Roots and New Trends in Logicism Workshop
Abstracts

Invited talks

Ludovica Conti (University of Vienna) - Arbitrary Frege Arithmetic

This talk focuses on a less-known version of Abstractionism, that we will call Arbitrary Frege Arithmetic. This system aims to restore Frege's Logicist project (i.e. a logical derivation of second-order Peano Arithmetic) by adopting, as much as possible, its original tools, namely a second-order logical system augmented with a (consistent) version of Basic Law V.

My preliminary aim consists in arguing that two of the main open questions of the abstractionist projects, i.e. consistency and logicality, have a common source in what we can call the Canonical account of abstraction, that is based on the unquestioned adoption of classical logic, classical metasemantics and of an *at face value* reading of the abstractionist vocabulary.

Arbitrary Frege Arithmetic arises precisely by a three-fold weakening of this account: classical first-order logic will be substituted by negative free logic, semantical indeterminacy will be spelled out in terms of arbitrary reference and the abstractionist vocabulary will be recast in order to admit a consistent revision of Basic Law V. The resulting system will recover the consistency and achieves the logicality of the abstraction.

The changes mentioned above are supported by philosophical and formal motivations that allow us to consider them completely compatible with the abstractionist spirit. In addition, the system we are presenting is triply weaker than the original Fregean one and, for this reason, it shows that *Grundgesetze* proposal was just too strong for the goals it was supposed to achieve.

Fernando Ferreira (University of Lisbon) - On the alleged failure of Frege's original programme

Frege's programme aimed at grounding mathematics in logic. The programme collapsed because Russell's paradox showed that Frege's framework is inconsistent. Frege, on learning of the paradox, freely conceded its devastating force and, at the same time, tried hurriedly to rescue his programme. To no avail, however. Russell's version of logicism, as well as the more moderns neologicism and/or abstractionism and the recent proposals based on modality, depart in important ways from Frege's original programme. In this talk, we recall that a consistent subtheory of Frege's system is able to interpret full second-order arithmetic. Second-order arithmetic is not all of mathematics (in particular, it does not include set theory) but the development of the programme of Reverse Mathematics has been showing that subsystems of second-order arithmetic are, cumulatively, being able to develop almost all of ordinary mathematics. The reason why the above mentioned subtheory of Frege's system is able to develop full second-order arithmetic is twofold: (1) Frege's logicism succeeds fully within the realm of finite sets; (2) Frege's framework admits impredicative quantifications.

References

1. F. Boccuni, Plural *Grundgesetze*, *Studia Logica 96(*2), pp. 315–330, 2010.
2. L. Cruz-Filipe, F. Ferreira, The finitistic consistency of Heck's predicative Fregean system, *Notre Dame Journal of Formal Logic 56*(1), pp. 61–79, 2015.
3. F. Ferreira, Zigzag and Fregean Arithmetic, in *The Philosophers and Mathematics*, H. Tahiri (ed.), pp. 81–100, Springer 2018.
4. G. Frege, *Basic Laws of Arithmetic*. Translated and edited by P. A. Ebert and M. Rossberg. Oxford University Press, 2013.
5. R. Heck, The consistency of predicative fragments of Frege's *Grundgesetze der Arithmetik*, *History and Philosophy of Logic 17*, pp. 209–220, 1996.

6. B. Jacinto, Finitary upper logicism, *Review of Symbolic Logic 17*, published online, 68 pp., 2024.
7. S. Simpson, *Subsystems of Second-Order Arithmetic*. Springer, 1999.

Salvatore Florio (University of Oslo) - Critical Plural Logic and Abstraction (with Øystein Linnebo)

Cantor defined a set as the collection into a whole of determinate and well-distinguished objects, objects that can "exist together". Although he recognized that not all objects can exist together in this sense, he did not provide a theory specifying when some objects can exist together. Without such a theory, his account of sets remains incomplete. We argue that a version of plural logic, Critical Plural Logic, fits the bill. As a logic of co-existence, Critical Plural Logic captures the essence of the familiar "stage theory" of set formation but is far more general and avoids a gratuitous reification of stages. We also argue that this plural logic can be of great aid to Frege and logicism, supporting a large class of abstraction principles. Critical Plural Logic emerges as a compelling and fruitful theory to develop key ideas of Cantor and Frege.

Sébastien Gandon (Clermont Auvergne University) - Russell's architectonic logicism

Logicism is the thesis that mathematics can be derived from logic. Many versions of logicism circulate and have circulated, each giving different meanings to the terms "logical" and "derivable"; and various criticisms have been levelled at each of these programs. But until now, little attention has been paid to the target of logicism, namely mathematics: what mathematics do logicists seek to derive? In particular, how do logicists deal with the difference between mathematical disciplines (such as, for example, the differences between arithmetic, algebra, geometry, etc.)? Do they consider that disciplinary demarcations within mathematics must also be logically derived, or do they consider, on the contrary, that this is not required?

My contribution will address this issue, and show that the ambition of Russellian logicism was not only to logically derive the content of mathematics, but also to account for its architecture.

Gregory Landini (University of Iowa) - Frege's Logicism is Antithetical to Whitehead-Russell Logicism

Whitehead-Russell Logicism defends a revolution within mathematics whose agenda was to demonstrate that abstract particulars (numbers, spatial figures) and specialized kinds of necessities (arithmetic, geometric) governing them are no part of any branch of mathematics. It holds that (revolutionary) mathematicians are studying relations (a priori) when they do mathematics. The revolution began with the non-Euclidean geometers and was advanced by mathematicians such as Cantor who, according to Russell, "revolutionized mathematics" in his progress toward revealing that only relational order and not quantity is relevant to mathematical studies of limits and continuity. Whithead and Russell did not invent the revolution. They defended it and revealed its strikingly new implications. Frege's Logicism rejected it, maintaining that numbers are abstract particular logical objects (in his technical sense of an objects as opposed to a functions which are essentially unsaturated extensional entities). Frege was clear that our access is to the logic of functions, and that we know logical objects (including the True and the False) only indirectly and by means of their being correlated with functions by a non-homogeneous function. This opens the question as to whether Frege accepted that the functions that are concepts depend for their existence on numbers as logical objects and that the True might be the number 0.

Hannes Leitgeb (MCMP) - Semantic Analyticity and Carnapian Logicism

In this talk I want to defend a Carnapian version of logicism about mathematics according to which mathematics is analytic. The respective notion of analyticity will be a semantic one. The main difference to more traditional versions of logicism will be an epistemic one.

Contributed talks

Iben Bollaert and **Cato Andriessen** (Ghent University) - From *Logik* to Logicism: Tracing the Roots from Hegel to Frege

This paper investigates the relationship between Frege's Logicism and Hegel's Logic through an extensive comparison of their views on the structure of definitions. We argue that revisiting Frege in light of Hegel's philosophy of language and mathematics is essential for illuminating the philosophical roots of the logicist project. Although Frege has been characterized as a strict critic of the German Idealist movement (Milkov 2015, 88), recent interpretations challenge this view by emphasizing Frege's historical and philosophical indebtedness to this tradition (e.g., Käufer 2005; Reed 2007; Hanna 2013; Hylton 2013; Milkov 2015). We further this discourse by examining how Hegel's and Frege's conceptions of definitions elucidate their respective notions of systematicity in a surprisingly similar way. This comparative analysis is underexplored if not absent in the scholarship of both authors.

Our aim is to revisit Frege's writings on definitions, focusing on the letters to David Hilbert, in which Frege repeatedly criticizes Hilbert's conflation of definitions and axioms (PMC, 34-38). This critique pervades Frege's entire body of work (e.g. GGA II §55-67, §68-85; CP 118-121, 273-284; PW 244) and is reflected in the growing literature on this topic (Boddy 2021; Horty 2009; Kremer 2019; Shieh 2008). More notably, recent discussions on Frege highlight the ambiguous function of the definition as being "an arbitrary stipulation" (PW, 211) on the one hand, while simultaneously providing the foundation of Frege's logical system on the other hand (Perrine 2021). We argue that a similar ambiguity presents itself in Hegel's writings on definitions. On the one hand, Hegel criticizes definitions as an immediate step in finite, theoretical and synthetic cognition ('theoretical mind' in Hegel's PoM), distinguishing them from axioms, divisions, and theorems (GL 708-713; EL §228-§231). On the other hand, Hegel suggests that his entire system must be viewed as a series of definitions of the Absolute (EL §87). Comparing both authors, we see an equivalence between Hegel's definition as the universalizing of the particular to make a cognitive system possible, and Frege's definition as the place where sign and reference are constituted together, enabling the propositional form of sense and grounding a logical system. We conclude by returning to the broader influence of Hegel on Logicism in light of these considerations.

Matteo de Ceglie (University of Salzburg) - Good Company, pluralism, and the foundations of mathematics

In this paper, I plan to assess the abstraction principles that form the "good company" of the Cantor-Hume Principle (see Mancosu (2016)) according to their consequences for the foundations of mathematics. The main goal of Frege's logicist program was to reduce arithmetic to logic, thus founding mathematics on logic alone (Frege (1893-1903)). However, his use of Basic Law V and of the Cantor-Hume Principle led to Russell's Paradox. After Wright (1983) showed that there were ways to salvage Frege's program, there was a resurgence of interest on abstraction principles, culminating in the proof that in some cases abstraction principles can be used to derive the axioms of PA_2 (second order arithmetic) from impredicative second-order logic without falling into Russell's Paradox (Heck (2011)). Mancosu (2016) argued that there are several of these "good" abstraction principles, and that this poses the "Good Company Problem", that is, the problem of choosing which, among all these abstraction principles, is the "best" or "right" one. In the same book, Mancosu argued that they are mostly equivalent, since they can all be used to reduce arithmetic to logic. In this talk, I argue that this is not actually the case, if we assess them against the goal of developing a foundation of mathematics. In particular, I show that only the Cantor-Hume Principle can be actually used as the stepping stone for a foundation of mathematics, since it is the only one compatible with full ZF. The other ones all give rise to weaker fragments of set theory (General Set Theory, from Boolos and Thomson (1987), and "euclidean" set theories, see Parker (2013)), that cannot be used as a foundation of (classical) analysis or geometry (assuming the most minimal set of axioms). Finally, I point to the fact that a possible solution that saves all the "good" abstraction principles from the foundational perspective is domain pluralism, as sketched by Sereni et al. (2023).

References

- Boolos, G., & Thomson, J. J. (1987). The consistency of Frege's foundations of arithmetic. 1987, 3–20.
- Frege, G. (1893-1903). *Grundgesetze der Arithmetik, begriffsschriftlich abgeleitet* (Vol. I-II). Pohle, Jena.
- Heck, R. G. (2011). *Frege's theorem*. OUP Oxford.
  Mancosu, P. (2016). *Abstraction and infinity*. Oxford University Press.
- Parker, M. W. (2013). Set size and the part–whole principle. *The Review of Symbolic Logic, 6*(4), 589–612.
- Sereni, A., Fogliani, M. P. S., & Zanetti, L. (2023). For better and for worse. abstractionism, good company, and pluralism. *The Review of Symbolic Logic, 16*(1), 268–297.
  Wright, C. (1983). *Frege's conception of numbers as objects*. Aberdeen University Press.

Antônio Mesquita (Autonomous University of Madrid) - The Boundaries of Logicism: Definability and Non-Denumerability

There is a clear sense in which the logicist project depends on the success of Hilbert's program: an axiomatization of mathematics from logical axioms depends on the possibility of axiomatizing mathematics from some set of axioms, whichever it may be. Hilbert's program, at least in its original formulation, was shown to fail by Gödel's incompleteness theorems. The failure of Hilbert's program therefore implies the impossibility of the logicist project, at least in the strong version listed in Tennant (2023). Church (1934) suggested that the problem leading to the impossibility of a complete axiomatization of mathematics is the same as the problem behind the Richard Paradox, namely, the representability problem, an issue concerning the mismatch between denumerable formal systems and non-denumerable mathematical entities. To solve the representability problem, it is necessary to review the commitment to non-denumerable entities of the semantics for the formal system chosen for the axiomatization of some branch of mathematics. As is well known, the formal incompleteness problem resulting from the commitment to non-denumerable domains of entities of standard semantics for second-order and higher-order logics can be solved with the adoption of Henkin's (1950) semantics. Despite that, Henkin's semantics is still committed to non-denumerability to a sufficient degree for the representability problem to arise. A solution to the representability problem can be given by placing a further constraint on Henkin's semantics taking a hint from Henkin's (1953) nominalistic semantics. The resulting semantics is committed to definability instead of non-denumerability. Like Henkin's semantics, this semantics makes way for an axiomatization of mathematics and the logicist project, but as an advantage, it also solves the representability problem. Furthermore, this semantics turns out to be closely related to the predicative systems that neologicists such as Heck (1996) advocate for carrying out the logicist project.

References

- Church, A. (1934). The Richard Paradox. *The American Mathematical Monthly, 41*(6), 356–361. https://doi.org/10.1080/00029890.1934.11987569
- Heck, R. K. (1996). The Consistency of Predicative Fragments of Frege's *Grundgesetze der Arithmetik. History and Philosophy of Logic, 17* (1–2), 209–220. https://doi.org/10.1080/01445349608837265
- Henkin, L. (1950). Completeness in the theory of types. *The Journal of Symbolic Logic, 15*(2), 81–91. https://doi.org/10.2307/2266967
- Henkin, L. (1953). Some Notes on Nominalism. *The Journal of Symbolic Logic, 18*(1), 19–29. https://doi.org/10.2307/2266323
- Tennant, N. (2023). Logicism and Neologicism. *The Stanford Encyclopedia of Philosophy* (Winter 2023 Edition). E. N. Zalta & U. Nodelman (eds.). URL=<https://plato.stanford.edu/archives/win2023/entries/logicism/>

Russell Wahl (Idaho State University) - Russell's Logicism and Poincaré's Anti-Logicism

Russell’s logicism was born from his critique of Kant and his exposure to Peano’s logic in 1900, and Russell’s further study of relations. One of his earliest critics was Poincaré, who argued against the position that mathematics was at bottom just logic. The main target of this attack was Bertrand Russell, although Poincaré also battled against Hilbert, Zermelo and Cantor. While at the time Poincaré’s views did not carry the day, there have been in more recent times some more sympathetic presentations of his position, by Gerhard Heinzmann, Michael Detlefsen and Janet Folina, among others. Much recent criticism has focused on Warren Goldfarb’s 1988 analysis which held that Poincaré was ultimately concerned with psychological issues. Here I examine what Russell’s concerns were in holding his logicism and what Poincaré’s concerns were with his objections. Russell’s concerns were different from those of other logicists and also of Hilbert’s, whom Poincaré tied to logicism. Poincaré argued for a key role for intuition in mathematical reasoning and had a disdain for modern formal logic. The key issues in this debate focus on intuition and the nature of logic. I focus on different notions of “intuition” and how these affect each point of view. As Michael Detlefsen pointed out several years ago, Russell and Poincaré often were talking past each other. Nevertheless, we can shed light on at least one driving force in logicism by focusing on their separate concerns.